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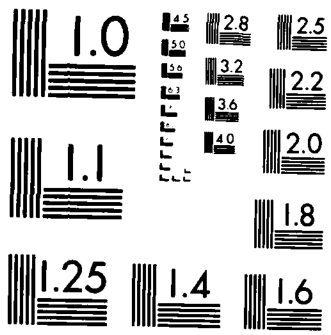

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REPORT SD-TR-85-25

AD-A155 988

Constrained Stiffness Matrix Adjustment Using Mode Data

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15 August 1984

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This report has been reviewed by the Public Affairs Office (PAS) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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FOR THE COMMANDER

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A procedure is introduced that uses, in addition to mode data, structural connectivity information to optimally adjust deficient stiffness matrices. The adjustments are performed such that the percentage change to each stiffness coefficient is minimized. The physical configuration of the analytical model is preserved, and the adjusted model will reproduce exactly the modes used in the identification. The theoretical development is presented, and the procedure is demonstrated by numerical simulation of a test problem.

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CONTENTS

TABLES

1. INTRODUCTION

Accurate dynamic models are required to establish the dynamic response of complex satellite structures. Unfortunately, analytical dynamic models of complex structures agree closely with properly measured mode data only in the first few modes. The effect of this deficiency can be minimized by using, as the dynamic model, the measured modes directly. This approach has been used successfully on numerous satellite programs. Another approach, generally referred to as system identification, is to adjust the analytical dynamic model in an attempt to improve correlation between analytical and empirical modes. A relatively large quantity of work has been published in this field. However, no procedure developed to date has achieved wide acceptance in the community.

Numerous goals and approaches to the identification problem are presented in the literature. For example, Rodden[1] published a procedure for establishing structural influence coefficients from modes of an effectively unconstrained structure. Subsequently, Hall,[2] using optimization theory, formulated a procedure that established a stiffness matrix such that the resulting analytical modes matched selected empirical modes in a least squares sense. The mass matrix was assumed to be exact. A procedure to adjust an analytical mass matrix, and derive an approximate stiffness matrix, was published by Berman and Flannelly.[3] Ross[4] introduced a procedure for deriving both

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[1]Rodden, W.P., "A Method for Deriving Structural Influence Coefficients from Ground Vibration Tests," AIAA J., Vol. 5, No. 5, May 1967.

[2]Hall, B.M., "Linear Estimation of Structural Parameters from Dynamic Test Data," AIAA/ASME 11th Structures, Structural Dynamics, and Materials Conference, Denver, Col., 22-24 Apr. 1979.

[3]Berman, A. and W.G. Flannelly, "Theory of Incomplete Models of Dynamic Structures," AIAA J., Vol. 9, No. 8, Aug. 1971.

[4]Ross, R.G., Jr., "Synthesis of Stiffness and Mass Matrices from Experimental Vibration Modes," SAE paper 710787, Sept. 1971.

the mass and stiffness matrices from measured natural frequencies and a square modal matrix composed of measured modes supplemented by arbitrary linearly independent vectors. A similar concept was recently published by Zak,[5] who supplements the measured mode data with information from analytically predicted modes.

Iterative procedures that employ statistical parameter estimation to adjust analytical models have been published by Collins, et al.[6] and Lee and Hasselman.[7] These procedures will preserve the physical configuration of the model. However, as demonstrated in Ref. 7, the agreement between the adjusted model and test data can, for some parameters, be worse than the agreement prior to adjustment.

Identification procedures based on matrix perturbation theory have been proposed by Chen and Wada,[8] Chen and Garba,[9] and Chen, et al.[10] These procedures assume that the analytical model parameters are close to the correct quantities. This is a relatively severe restriction since, for complex structures, experience indicates that at least a few parameters will not satisfy this requirement.

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[5] Zak, M., "Discrete Model Improvement by Eigenvector Updating," ASCE J. Engineering Mechanics, Vol 109, No. 6, Dec. 1983.

[6] Collins, J.D., et al., "Statistical Identification of Structures," AIAA J., Vol. 12, No. 2, Feb. 1974.

[7] Lee, L.T. and T.K. Hasselman, "Dynamic Model Verification of Large Structural Systems," SAE paper 781047, Nov. 1978.

[8] Chen, J.C. and B.K. Wada, "Criteria for Analyses-Test Correlation of Structural Dynamic Systems," J. of Applied Mechanics, June 1975.

[9] Chen, J.C. and J.A. Garba, "Analytical Model Improvement Using Modal Test Results," AIAA J., Vol. 18, No. 6, June 1980.

[10] Chen, J.C., et al., "Direct Structural Parameter Identification by Modal Test Results," AIAA paper 83-0812, AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics and Materials Conference, Lake Tahoe, Nev., 2-4 May 1983.

Identification procedures have also been developed using constrained minimization theory. Baruch and Itzhack introduced formulations to adjust analytical stiffness[11] and flexibility[12] matrices such that the resulting analytical dynamic model modes are identical to the analytically orthogonalized test modes used in the identification. These procedures assume that the mass matrix is correct. Berman[13] introduced a formulation that modifies the mass matrix and assumes that the measured modes are exact. Subsequently, Berman and Nagy[14] combined the mass matrix adjustment procedure of Ref. 13 with the stiffness matrix adjustment procedure of Ref. 11 to establish the so-called analytical model improvement (AMI) procedure. This procedure will yield a model whose modes agree exactly with those used in the identification. However, as demonstrated in Ref. 10, the analytical mass and stiffness matrices can be dramatically altered. Particularly troublesome is the modification of stiffness coefficients from values of zero to large magnitude nonzero values. Clearly, the introduction of load paths that do not exist in the actual hardware is undesirable.

As can be ascertained from the preceding discussion, a variety of approaches to the identification problem exist. For example, the statistical parameter estimation procedures assume that the measured modes and mass and stiffness matrices all contain errors. Some procedures assume that the measured modes are exact and adjust the mass and stiffness properties. Others assume the mass matrix to be exact, orthogonalize the measured modes, and then adjust the stiffness or flexibility matrix. Apparently, consensus exists only on the need to adjust analytical stiffness matrices.

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[11] Baruch, M. and I.Y. Bar Itzhack, "Optimal Weighted Orthogonalization of Measured Modes," AIAA J., Vol. 16, No. 4, Apr. 1978.

[12] Baruch, M., "Optimization Procedure to Correct Stiffness and Flexibility Matrices Using Vibration Tests," AIAA J., Vol. 16, No. 11, Nov. 1978.

[13] Berman, A., "Mass Matrix Correction Using an Incomplete Set of Measured Modes," AIAA J., Vol. 17, No. 10, Oct. 1979.

[14] Berman, A. and E.J. Nagy, "Improvement of a Large Analytical Model Using Test Data," AIAA J., Vol. 21, No. 8, Aug. 1983.

Analytical dynamic models of complex structures typically involve several hundred degrees of freedom (dof). However, at best, only a few dozen measured modes will be available. Thus, the number of stiffness coefficients will greatly exceed the number of equations provided by the measured mode data. It is reasonable to expect that a more accurate identification will result if the ratio of stiffness coefficients to available equations is reduced. This can be accomplished by supplementing the measured mode data with structural connectivity information. Of course, by doing so, we assume that the connectivity of the analytical model is correct. It is the purpose of this report to describe a procedure that uses, in addition to mode data, structural connectivity information to optimally adjust deficient stiffness matrices.

The stiffness, K, matrix adjustment (KMA) procedure will be developed using constrained minimization theory. An important part of the development is the selection of the error function to be minimized. In Refs. 11 and 14, it is suggested that the appropriate error function should involve the mass weighted difference between the analytical and adjusted stiffness matrix coefficients. However, in complex spacecraft structures, for example, non-structural items such as electronic boxes, batteries, and propellant generally account for over 80 percent of the total mass. Therefore, it is inappropriate to weigh the stiffness coefficient changes by any function involving the mass matrix. In addition, the error function, as defined in Refs. 11 and 14, can result in substantially greater percentage changes occurring in stiffness coefficients with numerically small values than in coefficients with numerically large values. This bias is difficult to justify.

Based on the preceding discussion, it must be concluded that the error function proposed in Refs. 11 and 14 is not suitable for our purposes and, therefore, a new error function must be established. It is proposed that the appropriate error function be independent of the system mass properties and stiffness coefficient magnitudes. Furthermore, minimization of the error function should minimize the percentage change to each stiffness coefficient. The optimally adjusted stiffness matrix can then be obtained by minimizing this error function subject to symmetry constraints, connectivity constraints, and constraints derived from the system eigenvalue equation.

2. THEORETICAL DEVELOPMENT

The preceding discussion indicated that it is reasonable to expect a more accurate stiffness matrix adjustment to result if, in addition to measured mode data, structural connectivity information is used. This can be accomplished by insisting that all coefficients with values of zero in the original stiffness matrix also have values of zero in the adjusted stiffness matrix. Mathematically, this can be achieved by insisting that the adjusted stiffness matrix [K] be related to the original stiffness matrix [k] by

$$[K] = [k]\Theta[\gamma] \tag{1}$$

where $[\gamma]$ is a matrix to be determined, and the operator Θ defines the following element operations:

$$K_{ij} = k_{ij}\,\gamma_{ij} \tag{2}$$

Therefore, if k_{ij} has a value of zero, K_{ij} will also have a value of zero. Since extensive use will be made of the element-by-element (scalar) matrix multiplication operator Θ, its relevant properties are presented in Appendix A.

To minimize unrealistic changes in stiffness coefficients, the error function to be minimized must be independent of the stiffness coefficient magnitudes (see Appendix B). This can be accomplished by defining the error function as

$$\begin{aligned} \varepsilon &= ||[\hat{I}] - [\hat{I}]\Theta[\gamma]|| \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} (\hat{I}_{ij} - \hat{I}_{ij}\,\gamma_{ij})^2 \end{aligned} \tag{3}$$

where

$$\begin{aligned} \hat{I}_{ij} &= 1 \qquad \text{if} \qquad k_{ij} \neq 0 \\ &= 0 \qquad \text{if} \qquad k_{ij} = 0 \end{aligned} \tag{4}$$

The adjusted stiffness matrix can now be obtained by establishing the $[\gamma]$ that minimizes ϵ and satisfies the following constraints:

$$-[M][\phi][\omega_n^2] + ([k]\Theta[\gamma])[\phi] = [0] \tag{5}$$

$$[\gamma] - [\gamma]^T = [0] \tag{6}$$

where

$[M]$ = system mass matrix (n,n)*

$[\phi]$ = system mode shapes (n,m)

$[\omega_n^2]$ = diagonal matrix of circular frequencies squared (m,m)

The constraint that the new stiffness matrix be consistent with the mode shapes and natural frequencies is introduced by Eq. (5). Inherent in this constraint is the requirement that the mode shapes exhibit the orthogonality associated with normal modes, i.e.

$$[\phi]^T[M][\phi] = [I] \tag{7}$$

The requirement that the new stiffness matrix be symmetric is introduced by Eq. (6).

The method of Lagrange Multipliers[15] will be used to incorporate in the minimization of ϵ the constraints defined by Eqs. (5) and (6). We begin by forming the Lagrange Function L

$$L = \epsilon + \sum_{i=1}^{n} \sum_{j=1}^{m} \lambda_{ij} \left(\sum_{\ell=1}^{n} k_{i\ell}\, \gamma_{i\ell}\, \phi_{\ell j} \right) - E$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} \mu_{ij}(\gamma_{ij} - \gamma_{ji}) \tag{8}$$

---

*Designates dimension of matrix

[15]Hadley, G., Nonlinear and Dynamic Programming, Addison-Wesley Publishing Company, Inc., Reading, Mass., 1964.

where λ_{ij} and μ_{ij} are the Lagrange Multipliers. The quantity Ξ represents the first term of Eq. (5), which is not a function of γ_{ij} and, therefore, does not need to be explicitly defined. Proceeding, we take the partial derivative of L with respect to each γ_{ij}. These derivatives are set equal to zero to obtain a set of nxn equations that the γ_{ij} must satisfy for L to be a minimum. Expressing these equations in matrix notation, we obtain

$$-2([\hat{I}] - [\gamma]) + [k]\Theta([\lambda][\phi]^T) + [\mu] = [0] \tag{9}$$

Equations (5), (6), and (9) represent the complete system of equations needed to determine the unknowns $[\lambda]$, $[\mu]$, and $[\gamma]$. Since $[\mu] = -[\mu]^T$, $[\mu]$ can be eliminated by adding Eq. (9) and its transpose

$$-4([\hat{I}] - [\gamma]) + [k]\Theta([\lambda][\phi]^T + [\phi][\lambda]^T) = [0] \tag{10}$$

Next, we pre-element-by-element multiply Eq. (10) by 1/4[k] and rearrange terms to obtain

$$[k]\Theta[\gamma] = [k] - 1/4\ [\Phi]\Theta([\lambda][\phi]^T + [\phi][\lambda]^T) \tag{11}$$

where

$$[\Phi] = [k]\Theta[k] \tag{12}$$

Substituting Eq. (11) into Eq. (5), we obtain

$$[A] + \{[\Phi]\Theta([\lambda][\phi]^T)\}[\phi] + \{[\Phi]\Theta([\phi][\lambda]^T)\}[\phi] = [0] \tag{13}$$

where

$$[A] = 4([M][\phi][\omega_n^2] - [k][\phi]) \tag{14}$$

Equation (13) can now be used to establish $[\lambda]$ which, when substituted into Eq. (11), yields the desired stiffness matrix [see Eq. (1)].

By performing the operations indicated in Eq. (13), and taking advantage of relationship 5 presented in Appendix A, it can be shown that

symmetric correction matrix orthogonalization procedure described in Ref. 16 will be used, i.e.

$$[\phi^c] = [\phi^m]([\phi^m]^T[M][\phi^m])^{-1/2} \tag{23}$$

where $[\phi^m]$ are the measured modes and $[\phi^c]$ are the orthogonalized modes. Only the first five modes were included in the orthogonalization, whereas all eight modes contributed in the response calculations. This simulates actual test conditions where modes that are not measured contaminate the measured modes. The two orthogonalized modes used in the subsequent stiffness matrix adjustments are compared in Table 9 to the "test" modes and the true normal modes.

The adjusted stiffness matrix obtained using the first orthogonalized "test" mode is presented in Table 10. Comparing these coefficients to the exact stiffness matrix (Table 1), it can be observed that many of the "analytical" stiffness matrix coefficients have improved substantially, particularly the diagonal terms. By comparing the adjusted matrix coefficients to those obtained with a single normal mode (Table 4), it can be observed that they are comparable. In addition, connectivity has been preserved.

The adjusted stiffness matrix obtained with two orthogonalized "test" modes is presented in Table 11. By comparing the adjusted stiffness matrix coefficients to those in Table 5, we observe comparable improvement to that obtained with two normal modes. In addition, as has been the case for all test problems, connectivity has been preserved. Thus, with a limited number of mode shapes, knowledge of the structural connectivity, and a physically realistic minimization function, it is possible to improve a deficient stiffness matrix.

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[16] Targoff, W. P., "Orthogonality Check and Corre tion of Measured Modes," AIAA J., Vol. 14, No. 2, Feb. 1976.

Table 8. Generalized Mass Matrix--Simulated Test Modes

	1	2	3	4	5
1	1.00	0.07	0.05	0.07	0.02
2		1.00	0.06	0.07	0.04
3			1.00	0.05	0.05
4		SYM		1.00	0.08
5					1.00

Table 7. Comparison of Natural Frequencies

Natural Frequencies (rad/sec)				
Corrupted Stiffness Matrix	Adjusted Stiffness Matrix			Exact Stiffness Matrix
	1 Mode	2 Modes	3 Modes	
24.769	30.665*	30.665*	30.665*	30.665
38.776	31.737	31.711*	31.711*	31.711
38.859	31.850	31.763	31.763*	31.763
31.529	33.694	32.362	32.362	32.362
41.783	37.317	34.194	34.193	34.193
42.015	39.443	35.561	35.560	35.560
44.811	41.393	38.789	38.789	38.789
54.841	50.590	46.412	41.888	41.888

* Modes used to adjust corrupted stiffness matrix

Table 6. Adjusted Stiffness Matrix Coefficients--Three Normal Modes

	1	2	3	4	5	6	7	8
1	1.5	-1.5	0.0	0.0	0.0	0.0	0.0	0.0
2		1011.5	-10.0	0.0	0.0	0.0	0.0	0.0
3			1110.0	0.0	-100.0	0.0	0.0	0.0
4				1100.0	-100.0	-100.0	0.0	0.0
5		SYM			1100.0	0.0	0.0	0.0
6						1112.0	-10.0	-2.0
7							1011.5	-1.5
8								3.5

The stiffness matrix coefficients identified with three normal modes are presented in Table 6. By comparing these coefficients to the exact values presented in Table 1, it can be observed that they are identical. Thus, with a subset of the system normal modes, and knowledge of the structure's connectivity, the exact stiffness matrix coefficients have been identified.

The natural frequencies associated with each of the adjusted stiffness matrices are compared in Table 7 to the exact values. The correspondence between mode shapes, and associated frequencies, was established by performing a cross-orthogonality check between the true normal modes and the modes associated with each of the adjusted stiffness matrices. As can be ascertained from the table, the natural frequencies of the modes used in the identification are reproduced exactly by the adjusted model. A comparison of corresponding mode shapes also showed exact agreement. Furthermore, for the one- and two-mode cases, the modes and frequencies that were not used in the identification also are in closer agreement with the true values. The three-mode case, as expected, provides exact agreement in all modes.

To be of practical use, the KMA procedure must yield reasonable results when imperfect test-derived modes are used. To study the procedure's sensitivity to imperfect data, the applicable test cases were repeated using simulated test modes. The test modes were obtained by establishing the closed form quadrature response of the structure to multiple sinusoidally varving forces (A critical damping ratio of 0.01 was assigned each mode.). The multiple force levels were adjusted to obtained "measured" modes whose mutual contamination did not exceed approximately 10 percent. The unit-normalized generalized mass matrix for the first five modes is presented in Table 8. The level of contamination is what can be expected from a properly performed mode survey test of a complex structure.

Before proceeding with the identification, it is necessary for the "test" modes to be analytically orthogonalized. For the purposes of this study, the

Table 4. Adjusted Stiffness Matrix Coefficients--One Normal Mode

	1	2	3	4	5	6	7	8
1	1.7	-2.1	0.0	0.0	0.0	0.0	0.0	0.0
2		1013.5	-10.1	0.0	0.0	0.0	0.0	0.0
3			1275.8	0.0	-198.6	0.0	0.0	0.0
4				1237.4	-178.6	-198.5	0.0	0.0
5					1237.5	0.0	0.0	0.0
6						1279.7	-10.1	-4.1
7							1016.1	-2.0
8								5.1

Table 5. Adjusted Stiffness Matrix Coefficients--Two Normal Modes

	1	2	3	4	5	6	7	8
1	1.5	-1.5	0.0	0.0	0.0	0.0	0.0	0.0
2		1011.5	-10.0	0.0	0.0	0.0	0.0	0.0
3			1110.0	0.0	-100.0	0.0	0.0	0.0
4				1100.0	-100.0	-100.0	0.0	0.0
5			SYM		1100.0	0.0	0.0	0.0
6						1113.2	-9.0	-3.0
7							1012.4	-2.3
8								4.3

Initially, it appears that all 16 stiffness coefficients can be identified from two normal modes. However, as illustrated in Fig. 1, dof 6, 7, and 8 involve a load path indeterminacy of three. Thus, to obtain exact identification, three normal modes are needed. If less than three normal modes are used, an adjusted stiffness matrix will be obtained that minimizes the error function [Eq. (3)] and satisfies the symmetry [Eq. (6)], eigenproblem [Eq. (5)], and connectivity [Eq. (1)] constraints. To demonstrate this, the "analytical" stiffness matrix was first adjusted using only one, then two, and finally three normal modes.

The adjusted stiffness matrix obtained using one normal mode is presented in Table 4. Comparing this matrix to the exact stiffness matrix, shown in Table 1, we observe that many of the "analytical" stiffness matrix coefficients have significantly approached the true values. In addition, connectivity has been preserved.

The adjusted stiffness matrix obtained using the first two normal modes of the structure is presented in Table 5. Again, connectivity has been preserved. In addition, the load path stiffnesses associated with dof 1 through 5 are in exact agreement with the coefficients presented in Table 1. Furthermore, the terms associated with dof 6 and 7 have also improved substantially.

This two-mode case illustrates an interesting feature of the KMA procedure. The procedure will identify exactly those stiffness coefficients associated with the part of the structure for which the load path indeterminacy does not exceed the number of modes used. The stiffness coefficients associated with the part of the structure whose redundancy exceeds the number of modes used will be adjusted to minimize the error function of Eq. (3) and satisfy the connectivity, eigenproblem, and symmetry constraints [Eqs. (1), (5), and (6)].

As discussed previously, three normal modes would provide the needed conditions to identify the stiffness matrix coefficients exactly. Note that regardless of whether three, four, or all eight modes are used, $[\alpha] + [\beta]$ will yield only 16 nonzero eigenvalues, and the resulting stiffness matrices will all be identical.

Table 3. Test Structure Corrupted Stiffness Matrix Coefficients

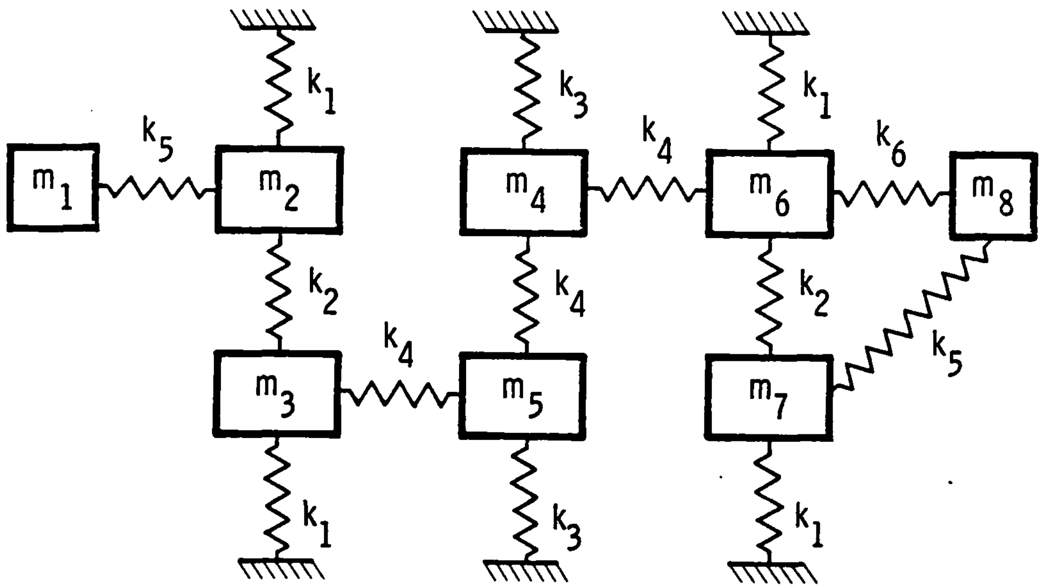
	1	2	3	4	5	6	7	8
1	2.0	-2.0	0.0	0.0	0.0	0.0	0.0	0.0
2		1512.0	-10.0	0.0	0.0	0.0	0.0	0.0
3			1710.0	0.0	-200.0	0.0	0.0	0.0
4				850.0	-200.0	-200.0	0.0	0.0
5					850.0	0.0	0.0	0.0
6		SYM				1714.0	-10.0	-4.0
7							1512.0	-2.0
8								6.0

Table 1. Test Structure Stiffness Matrix Coefficients

	1	2	3	4	5	6	7	8
1	1.5	-1.5	0.0	0.0	0.0	0.0	0.0	0.0
2		1011.5	-10.0	0.0	0.0	0.0	0.0	0.0
3			1110.0	0.0	-100.0	0.0	0.0	0.0
4				1100.0	-100.0	-100.0	0.0	0.0
5		SYM			1100.0	0.0	0.0	0.0
6						1112.0	-10.0	-2.0
7							1011.5	-1.5
8								3.5

Table 2. Test Structure Mass Matrix Coefficients

1,1	2,2	3,3	4,4	5,5	6,6	7,7	8,8
0.001	1.0	1.0	1.0	1.0	1.0	1.0	0.002



$k_1 = 1000$ $\quad$ $k_4 = 100$

$k_2 = 10$ $\quad$ $k_5 = 1.5$

$k_3 = 900$ $\quad$ $k_6 = 2.0$

$m_1 = 0.001$ $\quad$ $m_8 = 0.002$

$m_j = 1.0 \quad j = 2, 7$

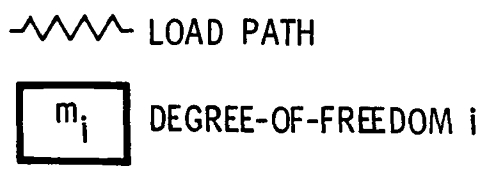


Fig. 1. Analytical Test Structure

3. DEMONSTRATION OF PROCEDURE

The KMA procedure will be demonstrated by numerical simulation of a test problem. The procedure will be used to adjust the corrupted stiffness matrix of an 8 dof analytical structure. The adjustments will first be performed using the normal modes of the system. The problem will then be repeated using simulated test modes.

A schematic representation of the structure is shown in Fig. 1, where the squares represent dof and the springs represent load paths. The stiffness matrix of the structure is presented in Table 1, and the diagonal terms of the diagonal mass matrix are shown in Table 2. This structure represents a severe test case because of the large relative differences in the magnitudes of some of the stiffness matrix coefficients. In addition, all eight natural frequencies are within 27 percent of each other, and the first four natural frequencies are within 5 percent of each other.

The stiffness matrix that represents the best "analytical" model of the structure is presented in Table 3. This matrix was obtained by modifying the stiffness matrix coefficients of Table 1. Some values were increased by a maximum of 100 percent, others were not changed, and two diagonal terms were decreased by nearly 30 percent. Comparing the two matrices, it can be observed that the "analytical" stiffness matrix does not resemble the true stiffness matrix, except for connectivity.

Before proceeding with the matrix adjustment, the number of independent coefficients in the "analytical" stiffness matrix will be established. As discussed previously, this can be accomplished by adding the number of diagonal terms in the matrix and the number of nonzero coefficients on one side of the diagonal. For the test problem, 16 independent terms are obtained.

$$[\psi]^T\{\bar{A}\} = [\psi]^T([\alpha] + [\beta])[\psi]\{\rho\}$$
$$= [\Omega]\{\rho\} \qquad (21)$$

where $[\Omega]$ is a diagonal matrix whose diagonal terms are the nonzero eigenvalues of $[\alpha] + [\beta]$. Using Eq. (21) to solve for $\{\rho\}$ and substituting into Eq. (20), we finally obtain

$$\{\bar{\lambda}\} = [\psi][\Omega]^{-1}[\psi]^T\{\bar{A}\} \qquad (22)$$

We can now construct $[\lambda]$ [see Eq. (17)] and establish $[K]$ using Eqs. (11) and (1).

The transformation defined by Eq. (20) yields a solution [Eq. (22)] that is applicable when the number of constraints defined by Eq. (5) exceeds the number of independent stiffness coefficients available for adjustment. If normal modes are used, the constraints will be consistent with each other and the true properties of the structure. Thus, the inclusion of additional modes, past a certain threshold, will not alter the identified stiffness coefficients.

This will not be the case if imperfect test modes are used. Because of measurement error, empirical modes will not necessarily be perfectly consistent with each other and the true properties of the structure. However, Eq. (22) will still provide a solution. Discussion of this feature is beyond the intended scope of this presentation. For the purposes of the present discussion, if we are dealing with test modes, we shall restrict our attention to the classical constrained minimization problem in which the number of constraints does not exceed the number of coefficients available for adjustment.

$$[\beta] = \left[\begin{array}{c|c|c|c|c|c}
[H]_{11} & [H]_{12} & \cdots & [H]_{1j} & \cdots & [H]_{1n} \\ \hline
[H]_{21} & [H]_{22} & \cdots & [H]_{2j} & \cdots & [H]_{2n} \\ \hline
\vdots & \vdots & \ddots & \vdots & & \vdots \\ \hline
[H]_{i1} & [H]_{i2} & & [H]_{ij} & \cdots & [H]_{in} \\ \hline
\vdots & \vdots & & \vdots & \ddots & \vdots \\ \hline
[H]_{n1} & [H]_{n2} & & [H]_{nj} & & [H]_{nn}
\end{array}\right] \tag{19}$$

where

$$[H]_{ij} = -\{\bar{\phi}\}_j\{D^j\}_i^T$$

$$\{\bar{\phi}\}_j = j^{th} \text{ column of } [\phi]^T$$

$$\{D^j\}_i = i^{th} \text{ column of } [\phi]^T[\hat{\Phi}^j]$$

Equation (15) can now be used to establish $\{\bar{\lambda}\}$. Note that the rank of $[\alpha] + [\beta]$ will not exceed a number equal to the number of diagonal terms in $[k]$ plus all the nonzero coefficients on one side of the diagonal. The exact rank of $[\alpha] + [\beta]$ will depend on the structural connectivity and the number of independent constraints defined by Eq. (5). For many problems of practical interest, $[\alpha] + [\beta]$ will be singular. In addition, $[\alpha]$ and $[\beta]$ are symmetric and $[\alpha] + [\beta]$ is indefinite; i.e., the eigenvalues of $[\alpha] + [\beta]$ can be negative, zero, and/or positive.

Equation (15) can be solved by first defining the following transformation:

$$\{\bar{\lambda}\} = [\psi]\{\rho\} \tag{20}$$

where the columns of $[\psi]$ are the eigenvectors associated with the nonzero eigenvalues of $[\alpha] + [\beta]$. Substituting Eq. (20) into Eq. (15) and premultiplying by $[\psi]^T$, we obtain

$$\{\bar{A}\} = ([\alpha] + [\beta])\{\bar{\lambda}\} \tag{15}$$

where the elements of $[A]$ and $[\lambda]$ have been written as column vectors $\{\bar{A}\}$ and $\{\bar{\lambda}\}$, respectively. For example

$$[A] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \longrightarrow \{\bar{A}\} = \begin{Bmatrix} A_{11} \\ A_{12} \\ A_{21} \\ A_{22} \\ A_{31} \\ A_{32} \end{Bmatrix} \tag{16}$$

and

$$[\lambda] = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \\ \lambda_{31} & \lambda_{32} \end{bmatrix} \longrightarrow \{\bar{\lambda}\} = \begin{Bmatrix} \lambda_{11} \\ \lambda_{12} \\ \lambda_{21} \\ \lambda_{22} \\ \lambda_{31} \\ \lambda_{32} \end{Bmatrix} \tag{17}$$

With $\{\bar{A}\}$ and $\{\bar{\lambda}\}$ defined by Eqs. (16) and (17), it can be shown that $[\alpha]$ and $[\beta]$ are as follows:

$$[\alpha] = \left[\begin{array}{c|c|c|c|c|c} [G^1] & 0 & \cdots & 0 & \cdots & 0 \\ \hline 0 & [G^2] & & 0 & \cdots & 0 \\ \hline \vdots & & \ddots & & & \vdots \\ \hline 0 & 0 & & [G^i] & & 0 \\ \hline \vdots & \vdots & & & \ddots & \\ \hline 0 & 0 & \cdots & 0 & & [G^n] \end{array}\right] \tag{18}$$

where $[G^i] = -[\phi]^T[\hat{\Phi}^i][\phi]$ and $[\hat{\Phi}^i]$ is a diagonal matrix whose diagonal terms are the i^{th} row of $[\Phi]$ [see Eq. (12)], and

Table 9. Comparison of Mode Shapes

	"Test" Mode Shapes	Orthogonalized "Test" Mode Shapes	Normal Mode Shapes
	0.029	0.112	0.144
	0.015	0.042	0.054
	0.374	0.356	0.360
	0.595	0.609	0.606
$\{\phi\}_1$	0.604	0.600	0.605
	0.356	0.365	0.362
	0.117	0.076	0.062
	0.551	0.522	0.505
	2.753	2.774	2.776
	0.909	0.917	0.915
	0.098	0.138	0.127
	-0.080	-0.078	-0.088
$\{\phi\}_2$	0.027	0.050	0.041
	-0.127	-0.123	-0.124
	-0.365	-0.330	-0.337
	-0.539	-0.498	-0.506

Table 10. Adjusted Stiffness Matrix Coefficients--One "Test" Mode

	1	2	3	4	5	6	7	8
1	1.7	-2.1	0.0	0.0	0.0	0.0	0.0	0.0
2		1030.4	-10.1	0.0	0.0	0.0	0.0	0.0
3			1276.6	0.0	-198.6	0.0	0.0	0.0
4				1235.2	-178.6	-198.5	0.0	0.0
5		SYM			1239.3	0.0	0.0	0.0
6						1279.8	-10.0	-4.1
7							1002.1	-2.0
8								5.1

Table 11. Adjusted Stiffness Matrix Coefficients--Two "Test" Modes

	1	2	3	4	5	6	7	8
1	1.5	-1.4	0.0	0.0	0.0	0.0	0.0	0.0
2		1010.9	-8.0	0.0	0.0	0.0	0.0	0.0
3			1091.0	0.0	-88.8	0.0	0.0	0.0
4				1098.1	-99.6	-99.6	0.0	0.0
5		SYM			1094.0	0.0	0.0	0.0
6						1113.5	-11.9	-3.1
7							1013.6	-2.4
8								4.4

4. SUMMARY

A procedure has been introduced that uses, in addition to mode data, structural connectivity information to optimally adjust deficient stiffness matrices. The procedure was developed using constrained minimization theory. The minimization error function was formulated such that the resulting changes to stiffness coefficients are a minimum. The resulting procedure retains the physical configuration of the analytical model, and the adjusted model exactly reproduces the modes used in the identification.

The stiffness matrix adjustment (KMA) procedure was demonstrated by numerical simulation of a test problem. The procedure was used to adjust the corrupted stiffness matrix of an 8 dof analytical structure. The adjustments were first performed using the system normal modes, of which only three were needed for exact identification. The test problem was then repeated, using simulated test modes, with excellent results.

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APPENDIX A

PROPERTIES OF ELEMENT-BY-ELEMENT MATRIX MULTIPLICATION OPERATOR

The derivation of the KMA procedure used an element-by-element matrix multiplication operator, $\odot$. Relevant properties of the operator are presented below:

A-1. $$[c] = [a]\odot[b]$$

defines $c_{ij} = a_{ij}b_{ij}$

A-2. $$[c] = [d]([a]\odot[b])$$

implies $c_{ij} = \sum_{k=1}^{n} d_{ik}(a_{kj}b_{kj})$

A-3. $$[a]\odot[b] = [b]\odot[a]$$

A-4. $$\{[d]([a]\odot[b])\}^T = ([a]^T\odot[b]^T)[d]^T$$

A-5. $$\underset{n\times n}{[a]}\odot\underset{n\times n}{[b]} = \sum_{j=1}^{n} [\hat{a}]_j[\hat{b}]_j$$

and

$$\hat{a}_{\ell k} = a_{jk} \quad \text{for } \ell = j \qquad \hat{b}_{\ell k} = b_{jk} \quad \text{for } \ell = k$$

$$= 0 \quad \text{for } \ell \neq j \qquad = 0 \quad \text{for } \ell \neq k$$

Example:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \odot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{12} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{21} & 0 \\ 0 & b_{22} \end{bmatrix}$$

APPENDIX B

ADJUSTMENT PROCEDURE WITH STIFFNESS COEFFICIENT MAGNITUDE WEIGHING

In the theoretical development of the KMA procedure, it was asserted that, to minimize unrealistic changes in stiffness coefficients, the error function must be independent of the stiffness coefficient magnitudes. This was accomplished by defining the error function ε as

$$\varepsilon = \| [\hat{I}] - [\hat{I}] \odot [\gamma] \| \qquad \text{(B-1)}$$

where $[\hat{I}]$ was obtained by replacing the nonzero coefficients of [k] with 1.0s.

It is a relatively simple task to explore the impact of incorporating, in the error function, the stiffness coefficient magnitudes. This can be accomplished by redefining the error function as

$$\varepsilon = \| [k] - [k] \odot [\gamma] \|$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} (k_{ij} - k_{ij}\,\gamma_{ij})^2 \qquad \text{(B-2)}$$

The symmetry and eigenproblem constraints are introduced in the minimization of the above error function, as before, using Lagrange Multipliers, to obtain

$$-2([k]\odot[k] - [k]\odot[k]\odot[\gamma]) + [k]\odot([\lambda][\phi]^T) + [\mu] = [0] \qquad \text{(B-3)}$$

We eliminate $[\mu]$ by adding Eq. (B-3) and its transpose

$$-4([k]\odot[k] - [k]\odot[k]\odot[\gamma]) + [k]\odot([\lambda][\phi]^T + [\phi][\lambda]^T) = [0] \qquad \text{(B-4)}$$

Proceeding, we define a matrix $[\bar{k}]$ such that

$$\bar{k}_{ij} = 1/k_{ij} \quad \text{if } k_{ij} \neq 0 \qquad \text{(B-5)}$$

$$= 0 \quad \text{if } k_{ij} = 0$$

Next, we pre-element-by-element multiply Eq. (B-4) by 1/4 $[\bar{k}]$ and rearrange terms to obtain

$$[k]\Theta[\gamma] \quad = \quad [k] - 1/4\ [\hat{I}]\Theta([\lambda][\phi]^T + [\phi][\lambda]^T) \qquad \text{(B-6)}$$

where all terms are as defined in Section 2.

Comparing Eq. (B-6) to Eq. (11), we note that they are identical except that $[\hat{I}]$ has replaced $[\Phi]$. Therefore, the solution as defined by Eq. (22) is applicable, and the only needed change is the substitution of $[\hat{I}]$ for $[\Phi]$.

The analytical test structure corrupted stiffness matrix (Table 3) was adjusted using one and then two normal modes. As with the KMA procedure, three modes provide exact identification, and both formulations yield exact agreement for coefficients defined by constraints only. The adjusted matrix coefficient, presented in Table B-1 (one mode) and Table B-2 (two modes) can be compared to the corresponding values obtained with the KMA procedure (Tables 4 and 5) and to the exact values (Table 1). As can be ascertained, the coefficients identified with the KMA procedure are in considerably better agreement with the exact values than the coefficients obtained using the error function defined by Eq. (B-2).

Particularly troublesome, with the error function defined by Eq. (B-2), are the large changes that occur in stiffness coefficients with small relative numerical values. For example, the diagonal term associated with dof 8 was modified from a value of 6.0 (Table 3) to a value of 116 [Table (B-2)]. This should be compared to the adjustment made by the KMA procedure which changed the coefficient from the value of 6.0 to a value of 4.3. The true value for this particular coefficient is 3.5 (Table 1).

The poor performance of the error function defined by Eq. (B-2) is due to the inclusion of the stiffness coefficient magnitudes. The minimization of an error function, such as defined by Eq. (B-2), will result in small percentage changes to large magnitude coefficients and relatively large percentage changes to small magnitude coefficients. Therefore, to minimize unrealistic changes in stiffness coefficients, the error function should be independent of the stiffness coefficient magnitudes.

Table B-1. Adjusted Stiffness Matrix Coefficients--One Normal Mode

	1	2	3	4	5	6	7	8
1	8.4	-20.0	0.0	0.0	0.0	0.0	0.0	0.0
2		1479.0	-75.0	0.0	0.0	0.0	0.0	0.0
3			1484.3	0.0	-316.8	0.0	0.0	0.0
4				1071.7	33.5	-275.4	0.0	0.0
5					1095.2	0.0	0.0	0.0
6						1544.4	-49.7	-97.1
7							1503.4	-33.0
8								75.6

Table B-2. Adjusted Stiffness Matrix Coefficients--Two Normal Modes

	1	2	3	4	5	6	7	8
1	1.5	-1.5	0.0	0.0	0.0	0.0	0.0	0.0
2		1011.5	-10.0	0.0	0.0	0.0	0.0	0.0
3			1110.0	0.0	-100.0	0.0	0.0	0.0
4				1100.0	-100.0	-100.0	0.0	0.0
5		SYM			1100.0	0.0	0.0	0.0
6						1278.0	135.0	-139.0
7							1137.0	-121.0
8								116.0

NOMENCLATURE

$[A]$	see Eq. (14)
$\{\bar{A}\}$	vector of [A] elements [see Eq. (16)]
$[\alpha]$	defined by Eq. (18)
$[\beta]$	defined by Eq. (19)
$\{D^j\}_i$	i^{th} column of $[\phi]^T [\hat{\Phi}^j]$
$[\gamma]$	matrix of stiffness matrix adjustment coefficients
γ_{ij}	element ij of $[\gamma]$
E	$[M][\phi][\omega_n^2]$
ε	error function
$[G^i]$	submatrix i of $[\alpha]$
$[H]_{ij}$	submatrix ij of $[\beta]$
$[I]$	identity matrix
$[\hat{I}]$	see Eq. (4)
$[\kappa]$	analytical stiffness matrix
k_{ij}	element ij of $[k]$
$[K]$	adjusted analytical stiffness matrix
K_{ij}	element ij of $[K]$
L	Lagrange function
λ_{ij}	Lagrange Multiplier

NOMENCLATURE (Continued)

$[\lambda]$	matrix of Lagrange Multipliers λ_{ij}
$\{\bar{\lambda}\}$	vector of $[\lambda]$ elements [see Eq. (17)]
$[M]$	mass matrix
μ_{ij}	Lagrange Multiplier
$[\mu]$	Matrix of Lagrange Multipliers μ_{ij}
$\odot$	element-by-element matrix multiplication operator
$\{\rho\}$	transformed $\{\bar{\lambda}\}$ [see Eq. (20)]
$[\phi]$	matrix of normal mode vectors
$\{\phi\}_i$	i^{th} normal mode vector
$\{\bar{\phi}\}_j$	j^{th} column of $[\phi]^T$
ϕ_{ij}	element ij of $[\phi]$
$[\phi^m]$	matrix of measured mode vectors
$[\phi^c]$	matrix of analytically orthogonalized measured mode vectors
$[\Phi]$	$[k] \odot [k]$
$[\hat{\Phi}^i]$	diagonal matrix whose diagonal elements are the i^{th} row of $[\Phi]$
$[\psi]$	eigenvectors of $[\alpha] + [\beta]$ associated with $[\Omega]$
$[\omega_n^2]$	diagonal matrix of circular frequencies squared
$[\Omega]$	nonzero eigenvalues of $[\alpha] + [\beta]$
$[0]$	null matrix

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